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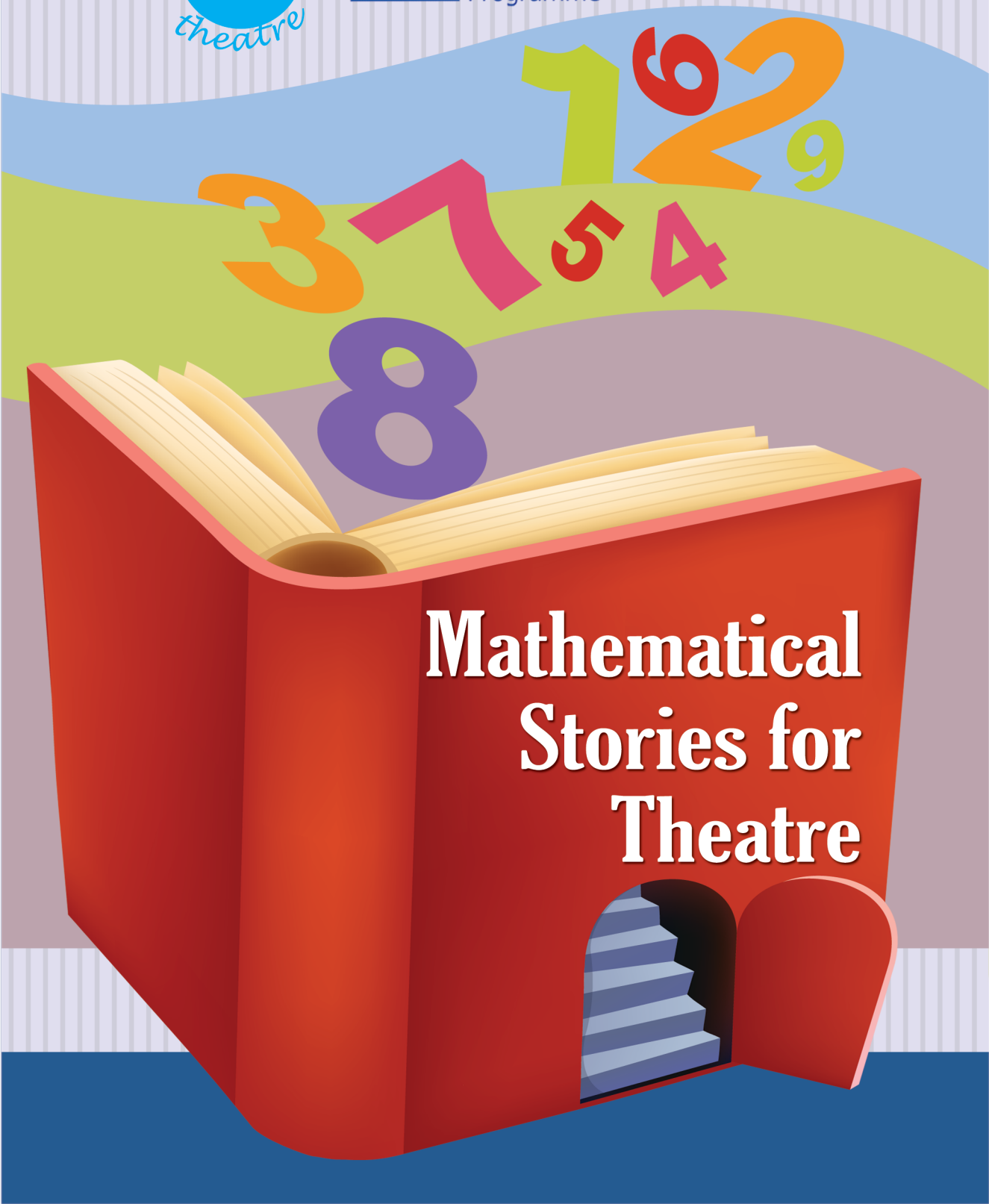
Lifelong Learning Programme



Mathematical Stories for Theatre

Lifelong
Learning
Programme

Le-MATH
Learning mathematics through new communication factors
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"Mathematical Stories for Theatre"

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Preface

A lot of people believe that mathematics is the first language the man had learned and this because he had learned to deal, basically he had learned to count, before he had learned to write. However, it is more important to accept that mathematical thought is the one that helps us to communicate easily and thus simply, because mathematics is the language of logic, truth and proof.

It's with great pleasure and satisfaction that I write the preface of "Mathematical Stories for theatre" that are now published for the Le-MATH project and I may say that this kind of story is new and helps the teacher's effort to give pupils a simpler and fancier way to understand the mathematical concepts and use it as a script for developing math theatre plays.

Our experience and knowledge seem to be the basis for continuing the effort of becoming better humans through our modern way of life, in spite of our position in society. Mathematics is the most important science, the basis of all knowledge.

I wish everybody, friends of maths, mathematicians, parents and pupils to love this book as it's a book of love for knowledge, a book of imagination that contains life-messages for better future.

I particularly thank the writer of "Mathematical Stories for Theatre", Georgia Charalambous, who accepted to write this and contribute it through the THALES Foundation to the Le-MATH project and to communicates with us through the dialogues of this book in a witty way, approaching mathematics in a different, innovative way, making the Mathematics teaching comprehensive and easy. I congratulate her for this reason.

Thales Foundation, as a partner in the project Le-MATH will continue to encourage the writing of new publications that give pupils and teachers new approaches of learning and teaching mathematics.

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1.

Elementary Operations
The children at Santa's Village

It was already morning when Nicol entered Perris's room and woke him up.

- Quickly, you must be ready, Santa has phoned. They had problems at the North Pole. We must gather all the children of the school to help him.
- And how will we go to the North Pole?
- He sends his reindeer, we must gather the children at the square, quickly!

Perri and Nicol started to wake up all the children of the school and gather them at the square.

- Have you put on your coats? It is cold at the North Pole, Nicol said.
- Yes, we have put them on, we are ready.
- Look at the reindeer; they are coming, look up at the mountain.
- Excellent! all the children shouted.

In a while, the reindeer came from the sky and took all the children to the North Pole.

The travelling was marvellous, snowy houses full of ornaments, snowy trees, joy and songs everywhere, Christmas atmosphere!

When they arrived at the North Pole, Santa Clause was waiting there for them... filled of love from all the children around, the dear able fat man with his red, uniform and his white long beard had blushed not from shame but from his anxiety for the preparation of the toys because there was a problem.

- Good morning, Santa, what is happening? asked Perri.
- Good morning Perri, good morning Nicol, good morning children, you know, there was a big catastrophe.
- What happened? Explain to us, said Nicol.
- A big avalanche had fallen on the factory and broke the north wall. The cold that entered inside, had frozen the machines. We managed to rebuild the wall, however the machines have been destroyed. I don't know what to do and we don't have enough time.
- Don't worry. We will think something, Nicol said.
- Let's go inside the factory to see the damage, said Perri.

They went to the factory where they saw the machines frozen.

- They can't work, said Santa, full of sorrow.
- Never mind, said Perri, whatever the machines had done, we'll do, Santa. You do so many things for us!
- Hurry up, all to work, shouted Nicol.
- This factory makes mathematical and educational toys. All toys will be gone to the children, tomorrow, on Christmas day.
- What are we waiting for, then, we must move the ribbons to be on time.
- And how will this be? asked Santa.
- We 'll take these twenty bicycles and join them on the ribbons. The children who are going to cycle, they are going to move the ribbons as well, like the machine did.
- Marvellous idea, children, says Santa with relief.
- What strange toys, observed Nicol.
- This factory makes toys that make the children smart .
- Really?
- Let's us have toys as well, shouted the children.
- Naturally, you will get some, says Santa laughing.
- What is this room? asked Perri.
- This room is the one of fabrications, answered Santa.
- What are they making?
- Don't you see? They take pieces of wood, metal and string and create toys that sharpen the brain.
- We, the children will do the easy work and the goblins will do the hard work. In this way we'll cooperate all together to catch up.
- O.K. then, let's start.
- The work that this goblin does, looks easy, let's go and ask this girl-goblin, said Perri.
- Could you tell us what are you doing here? asked Nicol.
- My name is Anna and my work is to count the toys. Other goblins put the toys on the ribbon. I must count every toy. I add to the number I have counted one more and I have a new number. My job is to make an addition in this way.
- We can do it as well, so go to help other goblins and Maria will do this, said Nicol.

All the children move to the next room where there were many goblins who were taking some toys from the ribbon. Perri moved on to a boy-goblin and asked him.

- Why aren't you with the others?
- My name is Michalis and I must be up here to do my job.
- And what is this job? asked Perri.
- Don't you see, I count toys. Other goblins see the defective toys and take them off the ribbon and I count every defective one. My work is the subtraction.
- Fascinating! But we can do it. In fact Costas can do it, to catch up.

Santa told them to go and see another rooms. They moved on and arrived at the storage room.

There Perri found a small goblin.

- Excuse me, what are you doing here alone? asked Perri.
- I must be very careful. I'm Yiannis and my job is to estimate whatever comes here. As you see, here there are twenty ribbons. Other goblins pass the toys from the ribbon and put them in the big one. I count every toy from every ribbon. I multiply the toys coming with the ribbons and I have the sum. My job is the multiplication of all the toys and ribbon.
- This is a difficult work, said Perri. However, Petros has practised on this. You can go and help the others. Petros will stay here!

After this, the children walk to the last room where many goblins put the toys in big boxes.

- Look, this is the last room, said Santa, looking at a goblin-girl.
- You little-goblin girl why are you doing this job? asked Nicol.
- As I am very clever, they put me here to do this important work!
- Really, what are you doing? said Nicol.
- The others globins take the toys from the ribbons and put them into boxes. I write down every toy that comes of the big ribbon. I separate the coming toys out of the toys that are put in every box and in this way I know how many boxes I'll need. My job is the division and I do it very well and carefully.
- Marvellous, shouted Nicol. We also have the cleverest student who could do your work. Her name is Kari and she is the best. It is very important all the mathematic operations to be done carefully to have right results.
- Fine, we have done whatever the machines were doing but we did it manually and also we have replaced many goblins so they could help the others.
- Yes, you succeeded, and now we'll catch up. I don't know how to thank you, says Santa full of joy.
- It isn't necessary, you have done so much for us.

- No, It isn't enough, I have to do something for you.
- It will be enough to continue to bring us present every Christmas and to see you from time to time on your sledge in the sky!
- You know, how much we love you, there isn't Christmas without you as there isn't Christmas without love, Jesus taught us.
- I thank you for saving the Christmas, said Santa.
- We thank you, too, Santa and whenever you need help, you know where to find us.
- Children, it's time to go, our parents will worry for us, said Perri.
- Bravo to you Perri and Nicol, said the children you always help those who need help, it is a worthy effort.
- It isn't only the effort that counts, children, but it is the helping and loving each other. This is the meaning of Christmas, love!
- Come here children, to embrace Santa!
- We love you, Santa.
- I love you too, my children, so I'll give you all beside the toys that you asked for Christmas, I'll give you an extra present for everyone.
- A mathematic, educational toy to become cleverer, nevertheless you deserve it!
- Hurray to Santa, called the children altogether.
- Ah, and something else children, I'll take you personally by my sledge, around the earth do you want that? , and later on to your homes.
- Don't forget, that as I am Santa, I can with magic, mathematic operations to stop the time, so I'll take you home in a minute, what are you saying?
- Excellent, hurray Santa, hurray Christmas, hurray to mathematic toys, hurray love!

2.

Straight lines and angles
Trupot the robot learns straight lines and angles

Once upon a time there was Trupot the robot, a mechanical little man up to a size of a ruler. His house was a toy-factory and a mechanic had built it up for children to play. He'll sell it to toy-factories you know, if it passes all the necessary tests to consider suitable and harmless to children.

Trupot was working with battery and he was handsome. His head was like a small ball, his belly like a square box and his legs and arms like thin wood that were supported by two thin bases like shoes. His speech was mechanical and he spoke slowly and by syllables. He walked also, slowly and mechanically making a step from time to time.

Though he walked a little, he was falling down later on, making unregulated movements and noisy grindings. It was something wrong with him there was a mechanical problem, so the mechanic who built it up, left it on the toy-shelf and forgot about him. He started making other robots.

An ordinary night however, after midnight, at the time toys become alive, Trupot, full of sorrow, started to tell his friends, the others toys, his complains; and that he was forgotten there alone.

- Have you noticed, my friends, what is happening? They left me here, without finishing my construction and they suppose to build other robots like me!
- Yes, it's true, answered a doll. I have heard also, that they were about to make other robots.
- They will make you, Trupot, at first and you don't work perfectly up to the last detail, continued a train.
- But what will happen, next? What am I going to do? I'll stay here on the shelf, without being useful in anything? And without the children play with me?
- It isn't nice, we must do something to help our friend says the car.
- Let us meet then to talk about it, say the toys altogether; to find a solution, O.K.?
- Yes, O.K., shouted all the toys together.
- Let everyone say his opinion, his idea and we'll see....., says Trupot thoughtfully.
- We think about going to the mechanic who had built you and speak to him and ask him to repair you, said the animals.
- But, that can't be done because we can't speak to men!
- You are right, we can't.

- May be, to whisper it in his ear while he'll be sleeping, says the smiling clown.
- What are you talking about? Will he listen to us? say the other toys angrily.
- Let's go away from the factory, all of us, so that he will have no toy!
- And if we leave, what's the point? say the cubes. He will build other toys.
- Yes, it's true.
- We must think something else, says the robot... hm..hm... why don't we ask the wise books of the library?
- Bravo! Why we didn't think it before? Books may help us.
- Quickly, take a book all of you and may be you can find something into its pages.

So, all the toys began to search for a solution inside the pages of the books until the proper book was found by Trupot. It was a book of mathematics.

Searching in the pages of the book of mathematics, he noticed a chapter that roused his interest.

- That is, he thought.
- What is this? What did you find?
- I found a chapter that'll help me to become a perfect robot and if everything become as I think, I'll make accurate and right movements.
- Read, Trupot, so that all of us hear what you have found!
- Listen! The title is "Straight lines and angles".
- Eh and what is it?
- Listen and you will understand.

The straight line is the line that has no endings, neither a finishing point nor a beginning.

Therefore, I must learn to walk in a straight line to have a right walking step like a robot, and not falling down.

The semi-straight line is the straight line that has a beginning but no ending and it starts from one point and after.

So, I must walk even I start from a point after; like the semi-straight lines.

Now a rectilinear segment is a part of the straight line that has a beginning and an end.

So, I must learn to walk in rectilinear segments.

That is, the toys say, you'll learn to move correctly and in correct distances.

- Yes, and not only this, listen…

Two semi-straight lines when they start from the same point, form an angle. This angle has two semi-straight lines for sides and for its vertex the common point of the two semi-straight lines.

By the angles I'll learn to move my arms and legs correctly and have the suitable openings I'll have, for example, an obtuse angle whenever I want to stretch my arm much over 90o .I'll have a right angle when I want to bend my arm to 90o exactly. I'll have an acute angle if I want to bend my arm more, below 90o, up to 60o degrees.

- Bravo Trupot. You found the solution to your problem inside the pages of the books of Mathematics. How nice! The toys were shouting happily because Trupot was happy as he'll become a robot with right and perfect movements.
- Books give us knowledge and help us to enrich our knowledge too.

My dear friends, when we study and be concentrated we'll learn many important things to become better ones, says the robot again and again to the toys and so it happened!

So, Trupot repaired his motor troubles and the mechanic who had built it, decided to make other toy-robots for the children because he was amazed by Trupot's change.

3.

Triangles
In the land of mathematic triangles

- Ss, sh, be quiet, children our lesson begins.
- What we call this form children? said the teacher, remind me!
- Triangle, madam! called all the children together.
- Bravo!! And why we call it "triangle"?
- Because it has three angles, madam!

Laughter and happy faces. Only Aristos wasn't laughing. He was looking sullen and he was grumbling all the time. He didn't like much the lesson of mathematics but what could he do? He sat there in his desk till the end of the lesson, and when the bell rang he went home.

Home, at last! He said to himself. "I 'll have lunch and then I'll play on my computer".

- How were you at school, Aristos? asked his mother after lunch.
- Oh, let me alone mum. We had mathematics again, today, and you know, I don't like it, I am bored with it.
- But everything we know depends on mathematics my boy, and they are very useful to all men, said his father seriously.
- Yes, yes, you have told me, but now I'll play on my computer, let me alone, I want to be alone for a while.
- So, he opened his computer to play though he was tired.
- Eh, Aristos, said a voice coming from the computer.
- What is happening? Aristos asked.

Looking at a triangle that was talking to him through his computer.

- But you are a triangle!

Can you talk, says our friend amazed.

- Quickly, come on. I must show you my country, we don't have much time.

So, Aristos without realising it entered the fantastic world of the computer with the triangle.

They were slading through a colourful frozen river, passing a long distance in few seconds, till suddenly they stopped!

- Do you know Aristos, I didn't tell you before, we have heard that you don't like mathematics much and we want to show you a part of our world so that you might change your opinion about us.

- We love people very much and we always help them to solve many of their problems and become cleverer, realising the importance of everyone's life on earth.

The universe is enormous and it is full of love and knowledge, especially the mathematic one.

Our friend, listen to all this with admiration; the view also was marvellous, colours everywhere, a triangle sun was shining all around among triangle mountains and trees of many shapes; even rivers, with uncountable numbers in their waters, were flowing from the slopes. How beautiful there were! He was thinking full of charm.

- May I introduce my friends to you, Aristos?
- Yes, of course I want so much to meet them.
- Here is my friend, the equilateral triangle.
- The equilateral triangle?
- Yes, it is called so as its three sides are equal.
- Here is the isosceles triangle because only its two sides are equal and the scalene triangle, because its three sides are unequal.
- They have very strange names, says Aristos laughing.
- We are very glad to meet you, the triangles said.
- I 'm, too! Aristos answered.
- They stay in the part of the "side", when you meet the others..
- Are there any others?
- Come on, my good friends. Here is Aristos!
- We are very glad to meet you!
- I 'm too, Aristos said.
- I am the acute triangle
- I am the obtuse triangle
- And I, who have brought you here, Aristos, I am the , rectangle triangle.
- I am glad to meet you, but why are you called so?
- Listen, the acute triangle is called so as it has a small acute angle below the 90o degrees, the obtuse triangle because it has an angle over the 90o and I, the rectangular one, because I have an angle 90o exactly.
- I 'm very glad, said Aristos. And where are you living?
- In the side that is called "angle".

- Ah, how, nice, our friend shouted. Now I understand, I understand the triangle – lesson our teacher told us; I didn't understand because I wasn't listening.
- I thank you very much, my rectangular one.
- I thank you, too, for becoming friends, but you haven't seen much, yet. Our world, the mathematic world is big, very big!
- Beautiful and full of knowledge.
- I hope you liked our little journey...
- Of course, I did!
- Let's go to visit our area, then the triangle-family. But I'll give you a present first, a triangle made of crystal. So, when you have a difficulty in maths and hold it tightly in your hand, we'll come to help you immediately.
- Oh, what good and gentle triangles you are!! A voice is coming to Aristos ear slowly-slowly.
- Wake up, Aristos, you are sleeping in front of the screen, his mother whispers.
- Mm…, Aristos yawns. I want to study maths, mum.
- Maths? But you are bored with it!
- Not now, mum, I like it and I' won't be bored with it, Aristos says, holding secretly his little present, his crystal triangle in his hand and he smiles!

4.

Plane shapes
Sophie at the land of plane shapes.

- I can't learn it in time, they are a lot, again. Aristos, Aristos, my brother, shouted Sophie.
- But how can you learn your mathematics so easily? I am in great difficulty and it is too much.
- What is too much?
- These mathematic shapes.
- Do you know, Sophie, I have a secret but I'll tell you because you are my sister. Will you keep my secret?
- Yes, certainly.
- Come and see. I have it closed in my drawer.

Our friend walked to his room and opens his drawer. He opens a box and...

- Ah, a crystal triangle! How beautiful is it! Where do you find it? It's shining a lot!
- But this is my secret, Sophie, it's a magic triangle.
- Really and what does it do?
- Look and you'll see.

Then Aristos closes tightly the triangle in his hand and immediately the triangle appears in front of him full of life.

- Ah, where do you come from? asked Sophie with admiration.
- From the land of mathematics naturally, said the triangle.
- This is my rectangular friend and he will help you, said Aristos.
- I'm glad to meet you, I'm Sophie, Aristos' sister.
- I'm glad too, how can I help you?
- Can you take us to your land to show the plane shapes to my sister? They are too many and...
- Don't ask, let's go. Hold my hand tightly.

A colourful circle appeared around them at once; they passed over a frozen slading and covering a great distance in seconds, they stopped at a forest.

There was nowhere such a strange forest. Trees had roots on triangle mountains and they had different plane-shapes. Every slope of a hill was better than the other and rivers with

uncountable numbers in their waters, were flowing down the slopes.

- How beautiful they are all! Sophie was thinking.
- Here is the forest of the plane-shapes, my friends said the rectangular triangle. Planes have only surfaces that fix exactly, without standing out inside or outside. The only difference is that in my land these are trees full of life but in your school you call them plane shapes. Every mountain slope here, has a special shape like the squares for example on which we are now. Moving on we find the rhombus, then the trapeziums, the triangles like me, the circles, the rectangular and their cousins, the rectangular parallelograms and other different shapes of rectangles.
- They are drawn very nice, in pretty, perfect rows, said Sophie and they have every colour you can imagine. But how can I manage to discriminate all of them?
- It isn't difficult; with a little imagination and thought. For example, the squares have four angles of 90o, the rhombus has four sides equal but not equal angles the trapeziums have two parallel sides and one small side from the other, the triangles have three angles, the rectangular ones have two pair of parallel sides but the one pair is smaller than the other, the rectangular parallelograms has four angles 90o and their two sides are bigger than the other sides, circles are plane shapes without angles, they are round and the polygons have many angles and sides.
- You tell them, so nice, my rectangular, I understand them all and they are so beautiful shapes!
- Let's walk to the forest of the plane shapes. They got tired after a while and after eating and drinking something at Rectangular's house, they left for home; it was the time of return.
- This journey was of great pleasure, my rectangular, said Sophie embracing the triangle.
- I feel the same, Aristos said. However we have to return home because our parents 'll worry.
- Don't worry, the time is passing by differently here. You weren't away too long. Before we go, I'll give you something my Sophie, a small present to remember where we have gone today. A special bracelet! A bracelet that has all plane shapes on it in different colours!
- It's out of glass and it's brilliant. It's the most beautiful bracelet I have ever seen, said Sophie full of emotion.
- For you, Aristos, a shape board.
- We thank you, dear friend. You may come and visit us whenever you like. We didn't give you a present though!
- Don't think about it! Your friendship is the best present for me. We are friends and that it counts. That, and the love, to everyone, let's go now, it's time to leave!
- So, they left through a colourful circle, full of joy and happiness.

5.

Curves
Curves at the Luna park

Ring, ring...the bell rang.

- Good morning children.
- Good morning madam.
- Are you ready to leave? The bus has come for our excursion. What are you saying? Are you ready to go?
- Yes, the children were shouting with enthusiasm.
- Come on, get on the line and don't forget your things.
- At once, all the children entered the bus and after the teacher called their names for presents, they took off.

Every schoolboy and schoolgirl was happy for visiting the luna-park and they were anxious to arrive there and have fun.

Without noticing it, the bus arrived at the parking-place of the luna-park and stopped. All the children get out of the bus immediately. What a beauty! There were games everywhere in different colours and music of modern songs was filling the air.

- Now, my children, we are in the luna-park and you must be careful, the teacher repeated. Remember what I've told you, when we were on the bus: first, don't stretch your arms out when you are on quick games; secondly, remember to put your belt, whenever is necessary and the men in charge say so. Finally, remember to have head and legs inside the spaces of the games where you'll be sitting. Don't stretch them out so that not to be hurt, O.K.? Do you want to ask anything?
- No, madam, we understood. We're O.K.
- Excellent! Let's go!
- It is the first time we'll be having a lesson at a luna-park, madam, Maria says.
- Yes, I know and I hope you like it, children.
- What our lesson will be? ask the children with wonder.
- Mathematics, of course!
- Mathematics?
- But how will this be? What's the theme and what is its relation to the luna-park?
- There is a great relationship to it, children. Because the luna park is full of curves. Our theme is the curves and today we'll have fun playing.

- It's a fine idea, madam, said the children smiling.
- Let's move on to catch that moving-ship. I'll remind you that the row of the ship is a curve and as it moves inside that space, passes through different points and gives it, its shape.
- Come on, get on the moving ship. It starts its journey in a while! Don't you see, how it's going up and down?
- How nice! Oh oh! The children shout with enthusiasm.
- This movement, children, is a curve. Our first game is over. Let's go to the next one. It's a bit further form here. They get on the big wheel slowly-slowly. The wheel was their next activity.
- The wheel as you notice, is a big circle and at every point we'll sit, the wheel 'll be drawing a circle. So, the curve that forms a circle is called circural curve.
- That wheel is very big, madam and I'm certain it draws a big curve, added Lisa.
- Yes, it is true and it has a nice view...

After a while, they move on to the train, the next activity.

When the train started to move on and it was whistling, all were very excited and they were singing happily. And when it was about to stop, their teacher told them that it was drawing an artificial curve. The railway was a curve line.

Moving on, they were passing through an artificial bridge made of rope.

- Hold tightly to pass the bridge, children said the teacher, because this bridge forms a big curve,out of the gravity of the earth. Do you notice how does it swing while are we walking?
- We have passed on the other side now, madam...
- Fine, do you want to go on, to come down the iron, circular staircase. There we can enter in another activity.
- Let's go, let's go, the children were shouting, full of excitement!
- Now, we are going down the staircase that is also an artificial curve because it was built by men.
- In the train we enter now, we'll go on with a curve which is forming a chain; for this reason our curve is called catenary curve. Be careful; as we said, it moves on now...
- Oh, oh!
- Did you enjoy it, children?
- Yes madam, shall we go to the little cars now?
- Of course, we'll go now, immediately!

- You can enter the cars in pairs but be careful, your car not to crash the others, because the one who collides with the other cars less, he wins the next turn!

Later on, the game was over again and the teacher asked:

- When you were in the cars turning here and there did you notice anything?
- Yes, Mrs Kate. We drove in curve lines and in equals one from time to time.
- Bravo, children and when we all were in the cars we were in a big closed plane , all the curve lines we formed, are called flat curves. There are many others curves you will be taught in other lessons. Do you want to sit and have lunch now, in that restaurant over there? It's open!
- We are hungry, said the children smiling.

After eating and drinking at the restaurant, the children were ready to return to their school.

However, their teacher, after lunch told them once more about the curves. She repeated that there are natural and artificial curves everywhere as well as many others. She asked them to mention some objects that look the same with curves.

At once, the children began shouting objects that come to their minds:

- A bridge
- A ribbon
- A garland
- A chain
- A necklace
- Cables of the streets
- The circular staircases
- Tracks of wheels
- The railway
- The small rivers
- A unique circle
- A rope for playing "snake and ladders".
- A knot
- The right and left turning curves that encircle a kionocranus on a Kariadis' head, at the Parthenon, added the teacher.
- Bravo, madam, the children shouted with laughter.

All together applauded and thanked their teacher for the marvellous journey at the luna park.

The sound of the bus horn is heard from a distance...

- It's time to go, children, the bus has just arrived and it began to rain-says the teacher.

- So, they got back to school very happy. They were all satisfied.

As the bus was moving on, the sun which appeared behind the clouds and the rain were an amazing view for our little passengers.

- A rainbow!

- Madam, madam, the curve of the rainbow, everyone called, full of joy!

A heavenly, wonderful and colourful curve. A shining and glittering beauty like their childish hearts!!!

6.

Perimeter- Area
The measure-area

Our story begins somewhere in the north Mexico, in a desert with its hot sun and intolerable heat. There where the temperature can be 50o Celsius in shade and on the contrary, It can be over 50o below the sun and it's hardly tolerable. Imagine, how difficult it is for an explorer to explore under such hard weather – conditions.

It's a difficult work to explore in the dessert, because there are many dangers besides the hot sun. There are wild snakes, lizards and dangerous insects. Don't forget the wind storms as well, out of earth and sand or the low temperatures at nights.

This hostile area of the desert was chosen by three explorers to pass through: their names are Adam, Sammy and Donn.

They were three modern explorers, in ordinary white caps on their heads for reflecting the rays of the sun; they had also sun-glasses to protect their eyes. They were wearing shorts, white T-shirts and athletic boots.

Having on their backs their bags and the necessary equipment for the exploration, they cross some distance in the desert. Soon, their quick steps become slow and they look tired and sweat because of the absolute heat.

- Oh, I'm tired, says Adam, in a slow whisper.
- So am I, says Donn hardly breathing.
- What are you saying, guys, do you want to stop for a while? Sammy asks.
- Only for fifteen minutes.
- So, little?
- Yes, to drink water from our flasks , rest a bit and move on before a sand-storm appears.
- There isn't an oasis, right?
- Right.
- We have still two hour's' walk ahead.
- Ah, so little? It seems like a century to me.
- Come on, don't complaint, we'll manage Donn!
- Don't forget to spread sun-oil again, on your faces added Adam. We got burn!
- Yes, really!
- Well, cheers!

- What a fresh water!
- To refresh a bit!
- Not to be thirsty!
- To fall in a lake for a bath!
- We didn't find the oasis map shown and do you think about having a bath?
- Yes, the heat is intolerable, what else to think about?
- Yes, you must think about moving on.
- Our break is over, Sammy! Let's go.
- From now?
- The fifteen minutes have passed and we shouldn't be out uncovered before darkness. It'll be cold, suddenly, and we'll get sick and don't forget that sometimes it rains hard in the deserts.
- O.K., O.K.! Let's go.

So, they move on when suddenly Adam sees some palm-trees and runs like a madman.

- Eh, eh, trees, shade, come on!
- Eh, where are you going?
- There is shade, he says, falling down, on the sand very disappointed.
- Oh, Adam! It was a reflection!
- Yes, you told me, now, after falling down, on the sand.
- I couldn't tell you earlier, what should I do?
- That means, we are close to an oasis.
- Move on, quickly.

Soon, they reached an excellent place. There were palm-trees, rich sprouting and water that was the most important. They run to drink water, get cool and rest under the tress, listening to the clear water passing by the valley.

- We'll put our tent here, Donn says.
- O.K.
- Draw the line for a perimeter of the tent and get it ready quickly.
- O.K. Sammy.
- Will you bring the bags here, Adam?
- Yes, of course.

- Estimate the area of the floor of the tent, to find out the covered-area we need.
- I'll compare the surface to that square pillow which is 30cm2.
- I have compared it and it is twelve times larger than the pillow. So, the area of the tent is 360cm2, Adam, isn't.
- Yes, it is so.
- Well.
- Have you heard anything?
- Sharp sounds...
- A sandstorm! Cover you faces quickly and look back over the hill for our refuse . We don't have time to set our tent and the wind is coming.
- Come on, I found something, calls Donn.

It's a hole in a cave; enter inside!

- Just on time! Fortunately we have our bags with us.
- Let's stay here, till the sandstorm passes by.
- We were lucky.
- What's this black door on the ground? Look!
- I don't know. We can open it.
- OK, I draw the bolt now...
- Oh, Oh, we roll in.
- What a dust!
- What is this place? Are you all right?
- Yes, besides a few scratches!
- What are all these mixed outlines that are drawn on the ground?
- I don't know.
- I have found something, children.
- What is it?
- These are two words engraved on the stone: a perimeter and an area.
- I found this bar with the inscription "answer".
- You mean, we must remove this bar after having an answer for every perimeter and area of these drawings.

- Yes, which is the drawing that is in front of the bar ?
- The square.
- To find out the area of the square, we multiply the one side with the other one. However the square has its sides all the same, the square equals with E=a.a=a2 and the perimeter is the side multiplied four times, that is π=4.a.
- Are you ready? I remove the bar.
- Be careful, the engraved stone is moving; which drawing is now in front of the bar?
- The rectangular – parallelogram came on!
- To find the area of the rectangular parallelogram, we multiply the length with the width, that is the area of the rectangular is E=a.b while the perimeter is the one side multiplied twice, plus the other side twice again, that π=2.a + 2.b. We have the same results whenever the parallelogram isn't a rectangular one.
- Are you ready? I remove the bar again.
- Be careful, the engraved stone is moving again! Now, which drawing has come in front of the bar?
- The triangle has.
- Fine, Donn.

The area of the triangle, it's the half of the area of the parallelogram. That is, we find it, writing the half multiplied area that the parallelogram has. So, the area of the triangle is: E=β.υ/2 and the perimeter is π=α+β+γ.

- Are you ready? I remove the bar once more!
- Be careful, the engraved stone is moving; which drawing is in front of the bar now? I hope that'll be the last!
- It's the rhombus and it's the last drawing.
- The rhombus has all its sides equals and its diagonals are divided vertically. That's why we find its area by multiplying its two diagonals and dividing out of two. So the area of rhombus is: E=δ(1).δ(2)/2. and the perimeter is π=4.α, that is the side multiplied four times.
- Are you ready, I remove the bar again!
- Attention, the stone with the inscription is moving...

A loud noise was heard and a door was opened. They moved on slowly-slowly and passing the door, what did they see? An old trunk with a lock! They broke it down, opened the trunk and oh! Jesus!! What was found there? A tool of counting the area, of a Greek measure area».

- It might invented by an ancient Greek mathematician who had lived here, thought Donn.
- May be, nobody knows. We found it, that's the point!
- Well, take it out of the case of the trunk to see it.

Immediately, after having it out of the truck, a door was open and an oasis was seen on the other side of the cave. It was the exodus of the cave!

They get out of it, with enthusiasm and telephone the authorities, for help!

Soon, the archaeologists of the area had arrived there and so had the assistance!

The measure area» was given to the archaeologists who, of course, delivered it to the authorities to be put in the archaeological museum.

They were congratulated by the authorities of the museum for this important mathematical instrument that had been found by them in the end. Their exploration wasn't made in vain as the measure area » was found because of it!!

7.

7. Sets
The most beautiful camping of the mathematicians

We are in a camping of mathematicians', one that combines summer-holidays and mathematical knowledge: that is pupils' rest and training of mathematics. It is certain that pupils learn better when they aren't tired and have nothing else in their minds.

It was an unusual camping, different from others; a camping without tents and without taking precautions from the dangers of the forest. It was a camping suitable to our modern times. Of course, we don't underestimate others' camping importance. However, this one offers rest, calmness and mathematical knowledge as well. It lies on the mountains, away from the noisy towns, in a village full of green high trees and fresh air. It was all the comforts of modern hotel-apartments and it's very beautiful.

There, pupils of many different countries have just arrived. They got off the bus and cries of joy and enthusiasm in different languages filled the air at once!

They had come from Europe, Asia, Africa, Australia, South and North America, even of places of the Antarctic.

- What a nice hotel! It has basket and tennis courts, football-fields and a swimming pool. It's ideal for holidays.
- What's your name? The children began to ask one other to introduce themselves, after packing their luggage in their rooms.

It was dawn and all the pupils went to the dining-room for breakfast. Later on they went to the open classrooms to have their first lesson that was the sets !

The lesson was in an international language for everybody to understand.

They were all very careful and they were listening to every word their expert teacher was saying.

A set is a group of objects that share common characteristics have similar properties. An example of a set is the letter of the word continent.

The objects which belong to a set are called "elements of the set .

The set of the word "continent", the letters: c, o, n, t, i, n, e, n, t are the elements of the above set.

Sets are written in capital letters of the alphabet and two brackets (square parentheses) ex. A= {c, o, n, t, i, n, e, n, t}

- They are also written in diagrams.
- With description of their elements.

- With the writing of their elements.
- With description and use of a letter (variable).
- If a set has only one element is called, one member set.
- If it doesn't have any elements, is called null.
- If it has an infinite number of elements, is called an infinite set.

Now if in the {c, o, n, t, i, n, e, n, t} we think of other things that regard this set , for example B {t, h, e, c, o, n, t, i, n, e, n, t, o, f, t, h, e, e, a, r, t, h} or C{t, h, e, c, o, n, t, i, n, e, n, t, o, f, t, h, e, a, n, t, a, r, t, i, c} then this set {c, o, n, t, i, n, e, n, t} is called superset or set report of the sets B and C.

If the sets have exactly the same elements then we say they are equal identical and we write the symbol of equality = for example A {c, o, n, t, i, n, e, n, t} and B {t, i, o, n, c, e, n, t} then the set A equals B.

Two equal sets are equivalent too and they are marked with this symbol ≈ A≈B.

However, the equivalent ones may not be equal.

It's necessary to say that the element or letter that is in the set belongs or not to the set like for example the letter r belongs to the set A but the letter f not, that is the letter f doesn't belong to the set A.

However, if the set B has fewer elements that there are also in the set A, then the set B is an authentic subset of A like the example below:

A={c, o, n, t, i, n, e, n, t} and B={c, o, n, t }

We mark B(A (B is authentic subset A).

There is a union of sets when we form a new set consisting of elements of the two sets and we say 'A union B'. We write every element one time only.

A={c, o, n, t, i, n, e, n, t}

B={e, n, t}

A union B= {e, n, t, c, o, n, t, i, n}

If we create a new set consisting of the common elements of the other two sets then it is called the intersection of sets A and B it is read A intersection B.

An intersection B = {e, n, t}

The children, after giving some examples of sets to their teacher and having answers to several questions they had, they returned to the restaurant to have lunch. The first lesson had already finished!

However, in the dining room, pupils of the same country used to sit and have lunch with their friends; they knew and trusted them because they came from the same country and had the same habits and culture. That was a problem and it shouldn't be continued!

The children should become friends and cooperate with each other, no matter the country they come from or the language they speak. They should try to understand one another, get some information about their countries and culture.

Fortunately, they started having acquaintances through games: tennis, basket, football, swimming. Even during the lessons they were trying to communicate but the problem was still there; in the dining room they were sitting separately, every country in a table, with their own people around.

Their teacher had noticed it and tried to find a solution. He had a brilliant idea!

He told the men in charge of the hotel to let the children have lunch outside, in the open air.

Chairs and tables were put in a big circle outside, at the football field and inside this circle they put more tables in seven small circles which were dividing each other and all together, the centre!

Later on, when the children came to have their lunch, they sat at the seven circles according to their country but some children necessarily sat at the intersections of the circles and some close the centre.

The teacher then, going to the centre said to them: "All you, children, be the earth's set of pupils. The ones being in the seven circles are the set of pupils of every country.

The ones being in the intersections of the circles, are the intersection of the set of pupils of the next countries of yours and altogether with the set of pupils of the next countries of yours, you are the union of the sets of pupils of the next countries of yours.

Now come on at the centre, all of you, next to the buffet for food...

Now you are the intersection of all the sets of pupils for every country.

So, there, schoolboys and schoolgirls from all the continents of the earth sat all together realising that all the pupils no matter how different they are, they have common elements so that to be together in all intersections of the sets.

After a while, they were enjoying food of different tastes, from all the world, in the big buffet that was set outside.

It was the most beautiful and richest buffet food for everyone.

It was the best and bluest sky!

It was the best mathematical camping!

This important man was standing a bit further touched and thoughtful. He may be singing "if all the children of the Earth..."

A lot of conversation, in different languages was taking place everywhere and there was of no significance who were you, what was the colour of your skin; you were a small subset in that set that is called Earth!

8.

The Cube
The water cube

It's Sunday, a sunny day.

- Good morning, children says the mother. Are you awake?
- Good morning, says the father, did you rest?
- Yes, "good morning" to both of you. We rest.
- Get ready and after breakfast, we'll go for a walk, to the sea.
- Really? How nice! I like the sea very much. Aristos, what do you say?
- I like it, too, Sophie. I collect pebbles at the seaside.
- I collect seashells as well. I hope to find some today. I didn't find a lot the previous time. Only two, three...

Soon, they were looking at the endless blue colour of the sea, feeling the salty breeze on their faces.

- It's very nice here!
- Yes, it's excellent!
- 'Fine', say their father and mother, "we are going for a walk now".
- We are going to look for seashells and pebbles said the children. See you later!
- OK, let's go!
- Let's throw away, into the sea, some pebbles to see whose pebblle will go further. Do you agree, Aristos?
- It's a very good idea.
- They began playing in the shore throwing pebbles into the sea till they got tired.
- I think we should rest a little and then begin searching for seashells and rare pretty pebbles, Sophie, O.K.?
- O.K. Let's sit on the sand now and we'll think about it.
- Were you well on Friday, at school, Sophie?
- Well, we had a very interesting lesson at maths.
- What was it about?
- The teacher told us about the cube.
- About the cube.

- Yes, He told us that the cube is a rectangular parallelepiped, which had twelve equal edges.

The cube has six bases which are equal squares between them and every angle of it is 90o. If we draw a line from an angle of cube which is below, up to the opposite angle high up, then we have the diagonal of the cube. And because all the edges of the cube are equal the mass equals its length on the cube.

- For example we count the length of the cube and multiply it by itself three times, to estimate the mass of a cube of water. The result will be the mass of the cube.
- Yes, you are right, Aristos.
- Sophie, do you want to dig the sand a little and make a cube?
- Of course, I do!
- We can water it to be solid and we can make it flat with a stick to be perfect!
- O.K.
- Our friends make a cube at once. They admire it and play with the sand till....
- Sophie, look I found something.
- What a pretty stone?
- Pretty? It's marvellous!
- It's a cube, a transparent one that shines in the sunlight.
- It will be perfect for my collection.
- Sure, it's so beautiful.

Suddenly, the cube slips out of his hands, raises over them becoming bigger and bigger till they find themselves into it!!

- We are in the cube!
- But how it happened?
- I have put you in the submarine cube!
- Who are you?
- "The water cube" from the land of mathematics.
- The land of our friends...
- They sent me to meet you and they salute you!
- We are very glad to meet you. Shall we visit them?
- We can reach the bottom of the land of mathematics using a submarine balloon. You can see your friends during the journey but from a distance, because we are running out of time.

They vanish at once into the light and again, they appear in the world of maths in few seconds!

They recognise familiar places; places they had visited before with their friends and they soon arrive at the mathematical sea. A clear one that is shining in a blue colour!

- It's so pretty, says Sophie but all of a sudden the submarine, with a dive, goes down to the bottom of the sea.

It was a magic bottom, with uncountable numbers, fish of different shapes and colours and rich sprouting!

There were also, strange stones, mountains, rocks and clear creatures that move and live in the water with a mathematical accuracy. Their environment was extra ordinary and it was of great significance the fact that they were all, happy!

There isn't such an environment at the bottom of our seas, on Earth, Aristos thought. Our sea is hardly blue because of the pollution. There are rubbish and oil in many spots of it and in certain places there are also toxic ones. Because of this, some kinds of fish are going to vanish or die. The food of the fish, plagton, is vanished too. Fish and other aquatic organisms are hungry and certainly not happy like here! A lot of fish go to other areas to search for food and so the balance of the universe is disturbed because of these kinds which are found in different places where they don't belong and so they die.

Even the salty water will vanish as the ice melt from the Green-house Effect and the overheat of the planet.

People will also be in danger if this situation will be continued, added Sophie.

- Now, you realised my friends, why I have brought you here!
- Yes, we did but what can we do?
- You can take care of the environment where you live and breathe and you can ask your friends to do the same!

You can be a good example to others, especially to the grown-ups who aren't so innocent as you are! If, for example, somebody throws rubbish into the sea or leaves them at the seaside and you saw him, you can explain to him why he shouldn't do that. You can do the same at home, at school, everywhere, to let these places clean; your places! You can also study, while growing-up, scientific books that great scientists wrote, mathematicians or philosophers. There are many ways to help your planet. You, children, are the future of Earth. But, first of all, you have to study maths because they are very important to all of us. All life depends on mathematics, the whole life of the universe.

- It's time to go back. We'll emerge on the surface, pass over the land of your friends, so you can say hello, to them.
- Hello, friends.

- Hello, we'll see you another time, children.
- Thank you.

When they reached the seaside again, the 'water-cube' vanished, leaving behind a small cube out of clear stone for Aristos and a sea-shell with the shape of a heart for Sophie.

They hold them tightly in their hands and run to embrace their parents.

They had passed a wonderful day by the sea and on their way back home, they remember the words of the water-cube: "All life depends on Mathematics; the whole life of the universe"!

9.

The sphere
A sphere of other dimensions

We are in the state of Florida, in U.S.A., far away in the district of the Atlantic ocean.

In a nearby city, not far away from the cape of Kennedy from where spacecrafts are launched into space, there is a family with two grown-up children at the age of 15: a boy called Shon and a girl called Tamara.

They were very good kids but they were often arguing; that was their big fault. However, this probably happens to everyone: to fight each other sometimes! Who is going to do his homework first and use the internet, who is going to go shopping, even, who is going to avoid doing housework or help: someone working in the garden or their mother who comes back home from the supermarket carrying shopping bags.

Today Tamara studies maths and thinks...

I must draw a sphere for geometry and starting from the centre, I must find the spots that have equal distance from the centre of the sphere, to find its surface. I'll need my geometric equipment, a pair of compasses and pencils. Let's start...

At school, we learned that the radius of the sphere is called the secure distance of the centre of any spot of its spherical surface and its chord passing through the centre, is called diameter and to find the length of the diameter, I double the radius! If I draw a straight line or imagine a flat surface , this straight line and the flat surface can divide or be tangent with the sphere or can't have any common spot with the glove according to the distance of its centre which can be equivalently bigger, equal or smaller from the radius.

Thinking the flat intersections of the sphere , I realise that they are always circles and if a plane surface passes through the centre, then the intersection is a circle with a radius equal to the radius of the sphere and it is called the biggest circle.

Now that I'm drawing it like our earth, it seems to me that this flat-surface passing the centre looks like the meridian of the earth. That's why they have called it meridian plane.

- "What are you doing" asked Shon, interrupting Tamara's mathematical thoughts!
- I study the sphere of mathematics what do you want?
- Well, I wanted to apologize for yesterday. My behaviour was bad. You may use the computer whenever you like. I wanted to finish a difficult project and I was in a hurry; that's why I didn't let you...
- Well, never mind. Let's forget about it.
- Surely?
- Surely. We are brother and sister and we can't always fight!

- Thank you. Do you want to go for a walk with me, to rest and forget our lessons for a while?
- Yes, I would love to!
- Where do you want to go?
- We can ask our father to give us a drive to cape of Kennedy today, to watch the launch of a spacecraft which is going to take place there. It will be crowded, of course!
- Wonderful! There were several months, we didn't watch one because of the bad weather.
- Ready?
- Ready!

Soon our friends were going to the cape by car and when they arrived there, they got off and moved on to the fencing area where they could watch the launch of the spacecraft.

- I'm anxious.

So, am I! It's a very difficult mission, the astronauts...

- Yes, they get information about our solar system, our Earth and other planets of our galaxy. And all this with danger; they risk their lives doing that.
- They may also inform us about other galaxies in the near future...
- The launch is starting: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 launch! Magnificent!
- Look! What a view! The whole sky was lightened by the fire that the tail of the spacecraft was leaving behind.
- In such moments, I feel embarrassed for man's achievements, of how many things humans had accomplished in few years.
- The Man may succeed to travel in other dimensions, in another space and time later on.
- Really? It'll be marvellous, if ever happens!
- It may remain only a dream...
- We can return home, now.
- Not yet. Our father didn't come. We can meet him down there at the road. Let's walk for a while.
- O.K. We can walk a bit further.
- What a place? It's very pretty. However, the wind is blowing into my hair.
- It's an open place for a long distance, that's why the wind is blowing very often.
- Thank God! It wasn't blowing on the time of the launch or they might have cancelled it.
- It blows hard and... look at that clear, bright sphere in front of us.

- Where does it come from?
- It's moving on here...
- I'm scared, Shon; hold my hand!

Suddenly, our friends disappeared and found themselves somewhere else, in another dimension, in another space and time.

- Shon, where are we? What happened? It's all so bright here and there's a heat everywhere. What is happening?
- We were brought here, in another dimension by the sphere, passing through an open hole.
- May be. We can walk forward and find out.
- There are people holding hands or working with a smile on their faces, children are playing and singing and everybody looks happy and beloved.
- Yes, it's very strange.
- It's like we are in the land of love, in another dimension!
- But, you are in the land of love. Welcome!
- We're glad to meet you! Who are you?
- My name is Love and this is the land of Love!
- We don't know, how we came here.
- You were brought here, by the sphere of other dimension.
- A sphere of other dimensions?
- Yes, there are passing globes that go to other dimension in my world and in other planets and in many galaxies. However, you didn't manage to have this technology on earth, yet.
- You have such passings on earth as the triangle of Bermudes for example and many others, but if you pass through them by mistake, you can't easily come back without the necessary technology. You can only return accidentally by the way you had entered, and under the same weather-conditions. But this, it's a rare thing to happen!
- What shall we do, now, asked Tamara full of despair.
- I'll show you around the land of Love and I'll help you to return home, later on.
- We thank you, Love, said the children.
- As you know, you are on the Land of Love, in a perfect atmosphere, a clean environment, at a place where people live in beautiful houses, having nothing to worry about as there isn't any poverty or war. Nobody fights or complaints about anything. They never get bored or be sick, as deceases have vanished due to the cure our scientists have found.

Everybody reads, learns, works, discovers and enjoys being alive. We are a family.

Both adults and children play smart games that develop intelligence and we keep discovering new things and going on to the future. It is so nice and everyone love one another, Shon and Tamara think.

- If our planet were like this, it would be more beautiful than ever.
- You must try harder to be like us, without evil and selfishness; you must help each other with all your love...
- Love is everywhere... Love is everything...
- It's like a sphere which is going round and round for ever, without stopping.

At once, Shon and Tamara returned home on our planet, within a sphere. Actually, they found themselves at the area of the cape, where they have been before and seeing their father and mother who had come to drive them home, they ran and embraced them.

It was an emotional moment for the family, a magic one. A moment of love that all people of this world must have. They must love each other. Love is everywhere.

Love is everything.

10.

The cone
The cone and Nic's construction

It was an ordinary morning and the bell of the Gymnasium rang as usually.

The pupils ran into the classrooms at once, for the first lesson, without delay.

- Good morning, the teacher of mathematics, said.
- Good morning, said the pupils.
- Are you ready for our lesson? We're going to revise some things today to be prepared for a test.
- Yes, sir! We're ready.
- Fine! We start.
- However, some pupils weren't listening to the teacher; they were disturbing the others who wanted to listen and they were talking as usually.

Nick was one of the pupils who wanted to listen to his teacher's words and he was trying in vain to persuade the others to stop talking into the class. At the end, the naughty students were dismissed by the teacher and sent to the headmaster for punishment!

- We can now speak about the cone, a geometric shape with a circular base, the teacher said with relief! The cone has an acute angle and a convex surface. Let's see it on your computer, children. Draw it, so that to see and study it better, he said.

Find a spot high up and call it, for example K. Then, draw a straight line down, up to the circular base you have drawn before. Draw, again some semi-straight line segments and your cone has been shaped!

Your central semi-straight line is called axis of the surface of the cone.

So, we have a surface of the cone if a semi-straight line, from the spot written on your computer, makes an angle with your central pivot and makes a whole turning.

The set of the spots being on the surface of the cone and the internal ones, is called cone and its vertex is the spot you have marked with the semi-opening you have given.

Now, draw another cone with an imaginary line, from the vertex, a part being between this vertex and a plane that divides vertically the central axis of the cone and you'll have a right cone.

However, if your plane surface divides the central axis indirectly in an oblique way, the equivalent part of the cone is called an oblique cone. You can imagine as well, that the intersection of the cone is like a circular right disc but in the intersection of the oblique cone it is an elliptic disc.

- You have to study all this for the test. We'll learn more about the cone in another lesson later on. Do you have any questions?
- No, sir, thank you.
- Well, O.K. Can you give me any examples of cone-shaped objects in our everyday life before leaving to study?
- Yes, sir, fine!
- Tell us, Nick, what can you draw on your computer that can look like a cone?
- I can draw a volcano with a cone-shaped crater and an Indian tent!
- Very good, who is going to try next?
- I, sir, answered Rania. I'll draw a cone-shaped hat of a fairy, of a magician and tops of castles where princesses live in fairy-tales!
- I'll draw a cone-shaped horn that men of the king's court used to have in old times and bells.
- A cone-shaped whirlwind, a paper cone where villagers used to put chestnuts or dried nuts, artificial cone-shaped trees for constructions, an ice-cream-cone and so many others you can think of, added the teacher. All of a sudden, the school-bell rang and the children went out. It was a break-time!
- Goodbye, children, we'll meet on the day of the test.

All the pupils went out of the classrooms for the interval. They eat and drink and rest before the beginning of the next lesson. Nick was walking among them; He was thinking about the bad behaviour of his fellow-students, trying to understand why they act like that: laughing at their teacher, at the pupils, at him! It was an intolerable situation and it should stop. It wasn't fair for the teachers or the students who wanted to learn and get as much as knowledge they can, so that to be important people one day in the future.

He should do something; but what?

He was walking alone, while the words of his fellow-students were coming to his ears: "stand by! The nerd is coming!" and they were laughing at him.

Their teasing was hard; their behaviour was also very bad. However, Nick ignored them once more and got into the classroom. He was patient as always and as soon as the lessons ended, he went home.

Waiting for the dinner to be served, he was thinking about all this: how much his teachers must suffer because of this behaviour, how patient were they; poor teachers! It's unfair!

Teachers try so hard to give us a piece of knowledge; what shall I do? I must do something quickly! Nick said to himself.

After dinner, he went to his bedroom to think once more about all this and to find a way to stop it.

After a while, he had a wonderful idea: he decided to invent an electronic, mathematical game that could only respond to sound-electronic commands which will be given by ordinary human voice. It will also require silence, team-work and mathematics! However, if there was any noise or disturb of any kind, the game could stop and one might start again from the very beginning! Of course, it will be given to other classrooms too, if there were good results: that is, silence in the classroom, no disturb or talking of any kind!

Without hesitation he started doing it on his computer.

He was trying to do it day after day, night by night until he succeeded! He did it at last! Now he wanted the headmaster's permission to use it in his class and later on, in the other classes too.

He arranged a meeting with the headmaster and he explained everything to him = every detail of his mathematical game! The headmaster was satisfied and gave his permission.

Nick was very happy and he arranged everything with his teachers; the day and the time the game was going to begin!

Finally, the testing day for the new game has come. Everybody is on his computer, ready to play; the loudspeakers are open, letters and numbers are written at the sides of the screen of Nick's computer. The game has started.

Let's see if the game will be solved with success! The pupils, one by one, were answering to every question or telling unknown words to the crossword puzzle, making a cone out of number or words. When, later on, every riddle of the cone has been solved, all the cones were combined, making a cone-shaped sphere like our earth; thousands of colours covered the screen, forming a colourful, imaginary Earth on every screen of the computers. It was magic!

The mathematical game of knowledge has been solved! There was silence everywhere.

All were touched and happy for the results.

Some pupils were crying out of joy. Everybody helped with his answers and knowledge of maths. They created a colourful Earth, the Earth of knowledge and this was a great achievement!

There was enthusiasm and a lot of applause everywhere. The pupils were cooperating with each other without teasing and disturbs. They have become friends.

They had all done well at the test and some of them got "excellent". Nick was one of them but he was also given the first prize for his construction, the mathematical game that succeeded to unite all the students of the school.

The headmaster, to thank Nick for all this, he arranged a special class for all, at school, for mathematical games or educated ones. Everybody was free to visit it and play on the computers there different mathematical or educational games. He also, gave the pupils the chances to participate in different competitions in many lessons at school and get

prizes. Even a classroom of constructions and inventions for pupils was set at school because of him! He was a great man.

There are several important people like him today who give children many opportunities to succeed in life.

"Knowledge is power", you must realise it, children, and try to be like these special educated people.

And who knows? You may be scientists one day!

11.

The cylinder
The small Eskimo and the cylinder

Our story takes place in the northwest of North America, very far away, in Alaska.

The inhabitants of Alaska are Eskimos, people of great endurance to the harsh cold who live either by cold rivers or on the ice in igloos. They fish in the cold rivers or hunt polar bears, reindeer or seals trying to earn their living.

The members of an Eskimo family has just awoken and after a while, the father decided to go fishing with his son. They put on their overcoats which are made out of leather and went out. It was very cold. They felt it besides their heavy clothes.

- Where are we going, father? Asked Shien.
- To the ice-lake, my child. We can find some fish there!
- O.K., father. Do you want me to bring all our equipment?
- Yes, you can bring the bag and I can bring the sledge with the dogs to set off.
- O.K. father, I'll go at once.

Soon, the father has the sledge ready and the dogs bark with anxiety, ready to go.

- Come on, jump in, we set off, he said.

They slide at once with their sledge to the "iced lake".

- Everything looks the same as usual: white snow and an endless area of iced-mountains everywhere. You hear nothing except the barking of the dogs and the grinding of the sledge!

Of course, cold wind is blowing too.

Thank god, it isn't a snowstorm. In half an hour they were there at the lake and they sat to rest and to eat something before fishing.

While they were resting, the father was thinking about the place where it was less dangerous to sit and go fishing without having the fear of an ice-crack and falling in the cold waters of the lake.

- You have found a small hillock, father, let's sit here, said Shien.
- OK, take also your fishing rod and stay here. I'll fish with my fishing spear, but you can't use it because it's like a sharp spear with a rope to catch the big fish. It's also dangerous.
- I know it, father; don't worry, I'll fish with my fishing rod.
- O.K. Shien, I'll move on, a bit further.
- Good luck!

- The same to you, my son ...

Sitting there, Shien was throwing his rod with a bait into the lake to catch some fish.

Soon he had caught his first fish and he was very glad. However, it was too small and so were the other later on! He wanted to catch some big fish. At least, some fish bigger than those!!

After a while, something big was caught on his hook and he started rolling it up with joy. But, what a pity! It was a barrel! He rolled it up and left it there, outside, on the iced lake.

It was a cylinder, like the one they had studied at the Eskimo school the previous day, at maths.

He remembered that the teacher had told them about it: It was a solid shape with two lid surfaces that were combined by a cylindery surface.

The word "ozone" was written on the one side of the barrel. At once he remembered that they had a discussion about the "hole of ozone" in the atmosphere of Alaska, in a science lesson. He decided to estimate the radius of the circle of the lid surface of the cylinder to find out the total surface area of the barrel. Then he could estimate the height of the cylinder to find the volume of it, as well.

When the finished his calculations, he realised that the gas inside the barrel wasn't enough to close the "hole of ozone" in Alaska.

He was unhappy but he was going on fishing, thinking how wonderful would be to have enough ozone to fill the empty space!

His father returned in a while,...

- What are you doing with this barrel?
- I found it in the lake; there is a word written on one side of it; the word ozone! However, it isn't enough to fill the "hole of ozone" father, Shien said.
- Really, my child, answered his father, the only solution to this problem is to reduce fumes that are spread all over the earth, our planet! Come on, it's time to return home; let the barrel there, where you have found it.

We could ask somebody to remove it far away from here and search it.

- O.K. father, let's go!

At night, when everybody was sleeping, Shien was awake, thinking again about it! That is, how beautiful would it be to have enough "ozone" to fill the empty space of the atmosphere...

However, a good fairy, the fairy of the Eskimos heard him and wanted to help him to fulfill his wish. What do you say? Would she fulfilled his wish?

Maybe yes, but not without our effort!!

12.

Pyramid
The spatial pyramid

All human beings have dreams.

We have small or big dreams, dreams of our imagination or unfulfilled dreams of our childhood. We sometimes hope to come true.

All the same, our friends from Huston of Texas estate of the U.S.A., dream to travel into space. Every day, after school, they study the universe or space: they watch spacecrafts launch into space from the launching pad of the space centre.

Mark and Tony are good children with great love for mathematics and computers. However, there is no way to travel to space without a spacecraft, is it?

So, one day after the lesson of science at the high school, the students were told to make a construction of their own invention. Therefore, they decided to make a space –simulator like a small vessel for two people only, something like a spatial chamber.

- Come on, Mark, we still have a lot of work.
- Have you brought all the equipment?
- Yes, we only have to fix them and test them, with our father's help.
- We must bring the chamber up to the time of the piston in order to not having any problems.
- It may be difficult.
- Difficult but not impossible, I dare say!
- We must do mathematical calculations, to be sure.
- Certainly, but it is late, let's go to sleep.
- Let's go as mother will be angry with us.

Opening the door of the big store to get out and before going on one-two steps they remained still, looking at the sky.

A celestial object was approaching them like a beam and it landed silently in front of them.

- Do you see what I see? asked Tony quietly.
- Ssss yes, and I can't believe it!
- It's an U.F.O. (an unknown flying object) in the shape of a pyramid.
- It is certain and I'm afraid...
- Don't be afraid, answered the alien who was inside the strange-vessel. I come in peace, I'm not evil.

- He is talking to us by telepathy. Let's listen to him and answer him by telepathy too.
- How do you come here?
- We, the aliens, make a lot of astral travels and sometimes we visit your planet, the Earth. I heard you talking and I know about the space-simulator.
- Yes, we want to have a spatial flight, that is to travel in space; to be astronauts. This is our dream!
- Do you want to be astronauts in my spacecraft for a while?
- If we are away for a long time, our parents will worry about us and we hardly know you how do you look like; we may be afraid of you!
- Come and see. I'ii get you inside! I'm simply very thin and light!

Then, in a gesture, in a magical way, he get them in the vessel.

- Don't worry. Time in space passes differently. So, you aren't going to be away more than a few minutes.
- But you look like a human being!
- Yes, but with developed intelligence and skills that you, humans, haven't develop!
- You can communicate by telepathy, if you want to!
- We can also speak many languages, like your language for example.
- How nice your spacecraft is!!
- It has big screens, a lot of buttons with a lot of different lights around them. It looks like a pyramid, I think. Why?
- But it is a pyramid! Do you know exactly what is a pyramid?
- According to what we know, when we have a flat base of any polygon and a mark outside of this base and combine these spots of the polygon, with the spot outside the base, we'll have a pyramid.
- The spot outside the pyramid is the vertex of the pyramid.
- The existed triangles are its bases, the polygon on the base it's also a base and its sides are called edges, continued Tony.
- The height of it is the distance of the vertex of the base, added Mark.
- This height gives stability to my vessel and with the shape of the pyramid we travel into space more easily! Look outside. Can you see how many manoeuvres we can do in space with this spacecraft?
- Good, but how we breathe here?
- This vessel is adjusted to give you the oxygen you need.
- How beautiful the space is! Endless and calm.
- It is full of stars, galaxies, meteors, comets and whatever human brain can imagine.

- Look at our planet, the Earth! It is blue, says Tony.
- Except those grey spots...
- Those spots are grey due to the pollution of the environment.
- Yes, it's true. I wish we would care more for our planet, Earth.
- You must care for your planet! It is important.
- Come to see how quickly we'll travel to other galaxies besides yours and how much speed we manage to accomplish to run fast. It's bigger than the speed of light. Come on, I'll also show you my planet!

In few seconds, they were in front of the alien's planet and they looked at it with fear.

- Is this your planet?
- It looks like our planet, the Earth! It is also blue!
- It is like the Earth but we live differently here. We don't have malices and wars. We live in peace and we look after one another. We have also develop a high technology here, one you don't have on Earth! Of course, we can help you to develop your technology as we have done for other civilizations.
- But how humans would accomplished such a development?
- First, humans must learn to love and help each other without being selfish. So, time will be creative because it will be used on inventions not wars. You must expand your knowledge in mathematics, science, medicine and with the power of Love you will succeed! There are still a lot of things you must learn! Come on, now, we must return to your planet, Earth.

In a minimum of time, they were back on Earth, at exactly the same place they had been before; by the door of the old store. They wished goodbye to their alien friend...

- Goodbye, friend, we'll never forget you.
- Goodbye my friends. We may be meet again one day in the future, when you will be grown ups and who knows, when you become astronauts.

Watching the pyramid moving away, they heard the door of the store banging with the wind. Trying to close it, they suddenly saw an amazing view: their construction was working by itself, like magic.

At school, later on, they got a big prize for their construction. They were also given a scholarship for this invention in a famous university.

They would study astronomy and mathematical science. They would be astronauts!

Their dream comes true!

My friends don't stop dreaming!!

13.

Prism
A meteor prism

In a city of Arizona of U.S.A....

An sms sound is heard...

- Eh, Willy, I've just had a message from Harry. He wants us to go to his house to study Maths for revision. Shall we?
- We'll go now Ken to have later on spare time for a walk.
- O.K.! wait to take my book. I don't want to forget it.
- O.K.

The children set off for Harry's house by their bicycles, so soon they were there.

- Welcome, children, Harry's mother said.
- Hello.
- Come on here, Harry calls from the dining room. There is plenty of room here to study the three of us. I hope you didn't forget your books and exercise books.
- Don't worry, we've brought them.
- Fine, my laptop is in my bedroom; I'll go to get it because we 're going to need it later, to look at different prisms. Have a seat, my friends!
- O.K. thanks.
- We have to say, what is a prism, Harry, before starting the revision.
- Yes, it's a solid geometric shape, with many bases(polyhedron); It's two bases are equal and parallel but all the others are parallelograms.
- You 're right, Willy. These two, equal and parallel bases of the prism, are called bases while the others are called side edges.
- The height of the prism is the distance between the bases and its edges that don't stand for the bases of the prism, are called side edges.
- Ken, can you remind us which is called a right prism? Read it for us!
- O.K.A right prism is called the prism of which the side edges are vertical on its bases. If they aren't vertical, it is called an oblique prism.
- A regular prism is the right prism which bases are regular polygons uses and we name them, according to the kind of the polygons we have;a square prism, for example, is the one which bases are square. If its bases are triangles, we name it a triangle-prism etc.

- Harry, the prism that has rectangular ones for bases, we name it a rectangular parallelepiped.
- Yes, and the total surface of the prism equals the area of every surface of it.
- The mass of a right prism equals the area of the base of the prism multiplied its height, adds Willy.
- Surely, says Harry. Let's see, now, on the computer several prisms and (let's us) solve some exercises for practice.
- You are right, the practice will make us better...

After finishing their revision exercises, they decided to go out.

- E, boys! Let's walk down to the plain to relax!
- It's the best hour for walking; it's afternoon and it's so beautiful! We can rest for a while, run and laugh...
- Well, Ken, shall we go? I'll arrive at the end of the plain first!
- I'll catch you up, Willy! I'll be the first...
- I'll pass you both, shouted Harry, running by quickly!
- Finally, Harry reached the end of the plain first, because he was thin. Few minutes later, Ken and Will, out of breath, arrived there, too.
- Let's breathe boys, says Ken laughing.
- We 're having a good time, here! What a beautiful place.
- We can see the little hillsides, the wild sprouting and bushes a far. The sun is now setting and it is a marvellous view!

Suddenly, they noticed something falling from the sky, forming a bright line like a shooting star.

Without hesitation and being ready for adventures the children moved on to the spot where the shooting star had fallen.

- It must be a meteor, Harry said. It's the first time I see such a burning ball or hear such a sound behind the hillside.
- May be, there are some remains of it, Ken said interrupting Harry.
- Certainly, but I think it will be small, if it was big we'll be on great danger.
- You are right, but, move on quickly to get it.

Soon, they arrived behind the small hills and they saw a stone, still steaming!

- It must be hot, still burning because of the friction while entering the atmosphere of the Earth whispers Will.

- We must be careful. We must wait for it to cold and then take it to our hands for examination.
- It got dark and our parents will worry for us.
- Don't worry, I'll telephone and tell them that we are on our way home. OK?
- OK! Thank god, there are the mobile phones! said Harry.
- Yes, it's true.
- It doesn't burn now, boys. Shall I take it?
- Take it, but carefully. It seems cracked this stone.
- How beautiful it is! It shines not only from outside but from inside too. There is something like glass in it.
- A cracked, spatial stone out of glass!
- Open the stone to see what kind of glass is it.
- It's like a well-shaped, solid, clear triangle-prism, out of glass, about 3cm.
- Look at its shape! It's wonderful!
- It's fantastic!
- It's a perfect prism, a solid geometric, symmetrical figure!
- What a discovery!!
- Come on boys! Take it to your hands and look at it!!
- It' s a shining prism, shouted the children raising their arms up out of joy!

While they are raising their arms high up, to the sky, the moonlight passing through the prism, gave at once a spectacular view.

Bright colours were emerging from the sides of the prism spreading colourful light all around!

The clear prism is given a refraction by the white light, even of the moon!!

The children, filled with enthusiasm were running home, holding in their hands a small treasure. A treasure they would only be hold for a while a s it would be delivered the next day to the scientific centre for studies.

However, they went there first and they were happy! It was their discovery and they felt great and important explorers. And who knows?

They may become great ones in the future!!

The purest and warmest feelings might emerge through the lighten prism of love and life!

For all of us! For all the world!!

14.

Equal Triangles – Uneven relations
A different lesson

- Good morning everybody!
- Good morning, madam!
- Do you remember what are we going to do today?
- Yes, madam. We'll have a different lesson, not from you but from us, the pupils.
- Precisely! All the pupils who were prepared for this lesson may come in front with their chairs and sit by the side of the classroom. The pupil who is going to start the lesson, may stay in the centre by the blackboard.

I'll sit with you at the back of the classroom like "a good school girl" behind a desk!

Some pupils laugh…

- We'll have a different revision lesson, in this way! A revision, about triangles. You will do it in your way, it will be your revision! Remember! I had given you the opportunity to choose the chapter you like most. Will you do well? We'll wait and see!!

Well, by this moment, you are going to teach not only your classmates but me as well.

- Your classmate is starting his lesson now!
- You know me, I'm Socrates, Mr. Socrates for today!

The pupils laugh…

- I'll speak to you, today, about the equality of the triangles.

You know, it's very difficult for all of us, to cut two triangles and put them side by side to find out their equality, every time we want to do so. It's better to prove it by information given in older times by great mathematicians with no doubts.

So, we have three theorems that prove the equalities of the triangles.

The first proof is when the three sides of a triangle are equivalently equal with the sides of another triangle. That is side, side, side and we mark them by three π (π-π-π).

I'll draw now on the blackboard two triangles. I name the one triangle ΑΒΓ and the other Α΄Β΄Γ΄.

Andreas, come here, please by the blackboard.

- According, to what you have said Mr. Socrates these two triangles are equal because they have equivalently their sides equal!

I write on the blackboard that ΑΒ=Α΄Β΄, ΒΓ=Β΄Γ΄ and ΑΓ=Α΄Γ΄. Therefore, as they have the

three equivalent sides, equal, triangle ΑΒΓ=Α΄Β΄Γ΄ according to the proof π-π-π (side-side-side).

- You are right Andreas. You may return to your seat.
- The second proof is when the two sides and the angle these two sides form, are equivalently equal with the two sides and the angle they had formed of the other triangle.

That is side-angle-side and we mark it Π-Γ-Π. Nicholas, may come here by the blackboard?

Draw for us, Nicholas, two triangles and write the second proof on the board to be seen by everybody.

- You can see from the triangles I have just drawn that the side ΑΒ=Α΄Β΄(equals Α΄Β΄). ΒΓ=Β΄Γ΄and the Β angle equals the other angle Β΄ (Β=Β΄). Therefore the triangle ΑΒΓ=Α΄Β΄Γ΄.
- Thank you, Nicholas...
- And now the third proof: when the two angles of a triangle are equal to the equivalent ones of another triangle and the side of the one is equal with the equivalent one of the other triangle, these two triangles are equal.

That is, side-angle-angle and we mark them by Π-Γ-Γ.

I'm drawing now, on the board the two triangles to see that ΒΓ=Β΄Γ΄and the angle Β=Β΄ and Γ=Γ. So we have: Π-Γ-Γ; therefore the triangle ΑΒΓ=Α΄Β΄Γ΄.

You may do now some exercises for revision.

- Do you have any question? The lesson is over!

They all applause.

- Fine, Socrates, the teacher said, smiling.

The next one may continue, please, she said.

- My name is Costas and I'll tell you today about the equalities of the rectangular triangles.

As you know the rectangular triangles have their angles equal, 90 degrees. Knowing that ,It's very easy to prove the equality of the triangles.

If I draw now, on the blackboard, two rectangular triangles ΑΒΓ and Α΄Β΄Γ΄I'll notice that angle Α=90ο and Α΄=90ο too. The other angle Β=Β΄=30ο. They are acute angles.

I'll also notice that side ΑΒ=Α΄Β. Therefore the two rectangular triangles are equal when they have ...Lisa, can you repeat for us the theorem?

- Two rectangular triangles are equal when they have an equivalent side and one angle equal.

- They must also have an angle of 90o as it is known.
- Bravo, you're right, Lisa.
- So, two rectangular triangles are equal when they have two equivalent sides equal.

Liza, can you give us another example?

Draw for us, on the board, as well, Lisa two rectangular triangles and tell us your observations.

- The data are the two rectangular triangles ΑΒΓ and Α΄Β΄Γ΄. The hypotenuse ΒΓ is 5 cm and the hypotenuse of the other triangle Β΄Γ΄5cm too. And the side ΑΓ=4cm like Α΄Γ΄ I have to find out if the triangle ΑΒΓ=Α΄Β΄Γ΄... I write ΑΓ=Α΄Γ΄= 4 cm (Π).

  ΒΓ=Β΄Γ΄= 5 cm (Π), ΑΒ=Α΄Β΄ (Π) according to Pythagoras' theorem about the rectangular triangle. Therefore ΑΒΓ=Α΄Β΄Γ΄.

- Very good, Liza, you may return to your seat,

Do you have any questions? If not, the lesson is over.

Nice, very nice, said the teacher, applauding with the rest of the pupils.

The last one, who is going to speak to us, is Marios.

- Yes, I'll tell you about the properties of the isosceles triangles.
- In an isosceles triangle, its two sides are equal all the same the angles that are by the bases!
- If an isosceles triangle draw a bisector or the height or a diameter to the base we'll observe that they are the same.

For the example, we draw an isosceles triangle ΑΒΓ. The data are: the isosceles triangle, the side ΑΒ=ΑΓ and the ΑΔ intermediary.

We want to know if the triangle ΑΒΔ=ΑΓΔ.

- Comparing the two triangles we find that:

1. The side ΑΒ=ΑΓ (Π)
2. The side ΒΔ=ΔΓ (Π)
3. The side ΑΔ=ΔΑ as it is the same side (Π). Therefore the triangle ΑΒΔ=ΑΓΔ.

For this reason, we presume that having the triangles equal we'll also have the other elements equal too, because the ΑΔ isn't only the bisector of the triangle but its height as well. The angles Β and Γ by the side of it, are equal too.

Do you have any questions? Well, I don't have anything else to add.

A lot of applause is heard from the classroom

- Bravo, bravo children. You have done very well.

Do you have any question about all of this? About what have we learned till now?

- Yes, madam…
- Well, What do you want to ask, Thalia?
- If we have a triangle with unequal sides, what is happening with their opposite angles?
- Is anybody here who wants to answer?
- Yes, Paul, tell us…
- If we have a triangle with unequal sides, then its angles will also be unequal and on the contrary! (vice-versa).

You've told us, madam, when we learned about the uneven relations of the triangles in previous lessons.

- Another question? I ask you, if you remember…
- If we have two triangles and the two sides of the one are equal with the two sides of the other but the angles that are inside these sides are unequal, what do you observe?
- Yes, Dino?
- We realize that the three sides of the triangles are unequal and on the contrary,
- Bravo!!
- Well, do you understand everything?
- Yes.

A lot of applause is taken place!

- What's your opinion about the fact that your classmates taught us like teachers?
- It was very good, Mrs Maria, shouted the children all together!
- You, pupils, who have taught us today; do you want to say anything? the teacher said.

Was it easy, being teachers?

- It's difficult to get everybody's attention, said Marios.
- Yes, you must always be well prepared and ready to answer questions to make the lesson interesting so that students stay focuses pay attention added Costas.
- It's a difficult job, indeed. However, you feel nice because you offer to your students the light of knowledge, added Mrs Maria.
- Teachers must study at home to be well prepared for their lessons. Good teaching needs preparation. It's a spiritual, tiring job, said Socrates. For example, I was well prepared at

home to be good today at the lesson; I was a teacher for a while! I don't know what do the others think about it, but I think your job is difficult, madam and we must respect you more!

- We thank you, mrs Maria for being our teacher, said all together. We thank also all the teachers for being so good with us. They give us knowledge and advice to be better people. You are our second family.
- I thank you, also, children, for today's lesson. It was really a different lesson, a special one. I also thank you for your good words…

I'm certain that with good behavior and a lot of knowledge for maths or of any other lesson you'll be not only clever but better people one day!!

15.

Pythagoras' theorem
Ancient, Greek, Mathematical museum

It was a fascinating night, at the Greek sky...

A night, full of stars that were shining in the endless black darkness up there in the sky.

The friend of our story was looking at them secretly and with fear while she was reading the last lesson for the next day.

Eleni loved maths very much and a revision about it makes her happy.

- Let's see if I remember... for the test tomorrow...
- The Pythagoras' theorem stands for rectangular triangles only. A rectangular triangle has two vertical sides the ones that form the 90o angle of the triangle, and a side that is opposite of that right angle and it's called hypotenuse of the triangle.
- If I draw a rectangular triangle ΑΒΓ and form three squares that each one have a side equal with one side of the triangle them I observe that here stands the theorem of the ancient Greek Pythagoras. That is, in every rectangular triangle, the square of the hypotenuse is equal the sum of the squares' vertical sides of this triangle.

  $(\text{ΑΓ})2 + (\text{ΒΓ})2 = (\text{ΒΓ})2$
- Therefore, this important theorem helps us find one side of the sides of the orthogonius triangle if we know the other two!
- I think, I remember them well and I'm sure I'll get on very well in tomorrow's exam.

Eleni was thinking about all this, closing her book and exercise book and tidying her geometrical instruments on her desk will all the rest things of her equipment.

In spite all this, she continued looking thoughtfully at the stars of the night.

They were so beautiful that she decided to go to sleep with the window of her bedroom wide open. She was looking at them with admiration thinking that if accidentally a shooting star was falling down, she could make a wish!

Having the lights of her room off, she was looking at the shining stars again and again.

In her mind, every star was a piece of knowledge or one of the mathematic theorems that were inherited to us by our ancient, wise ancestors like a spiritual inheritance.

All of a sudden, a star with a colourful tail behind, approaches her in quick circular movement and Eleni, without noticing it, found herself among the stars in the sky.

Crossing the endless sky, like a hologram on this bright star, Eleni, reaches an unknown, strange state in an unknown planet.

It was a clean state with a blue sky and a sea with such a blue colour that no one has ever seen.

There were people all around like holograms with beards and ancient clothes like the ancient Greek.

- What happened? Eleni, wondered, saying nothing. Am I at Ancient Greece? All these must be in a museum, I think.
- But you are, in a museum. This state is an ancient Greek mathematical museum!!!
- Who spoke?
- "The ancient Greek star", I have brought you, here!
- But you gave an answer to my thoughts. I didn't really ask you.
- We speak by telepathy here!
- This is great! Speaking by telepathy!!
- Yes, but it is even more important to see the things you have dreamed of, from very near! Let's go...

So, in fast speed they found themselves in the astonishing rooms of the museum!

- There were Greek displays everywhere: ancient statues, pots and more important Greek letters and mathematics on ancient papyruses.

There was a room for every ancient, wise mathematician, his theorems, inventions and more important, his hologram was there to welcome you with a sweet smile!

- You may ask whatever you like, Eleni, said the "ancient Greek star".
- Around every Greek mathematician there are holographic boards and you can ask anything you like about the Greek civilization or mathematic theorems. Every question will automatically be written on the board and it will be answered by the ancient mathematician himself; the one had invented and proved it.
- However you must press the button that stands for Modern Greek because here we speak only ancient Greek.
- Really? This is wonderful! We'll have such boards in the future to learn many things better!
- Yes, indeed in the near future!
- I'm impressed, my kind star, look... Pythagoras, Efcledes, Thalis from Meletos, Aristotelis, Eratosthenis, Platon, Archimides, Ippocrates, Diofantos and many others. And I may ask them many questions; how they succeeded to become wise and important people!

It's so nice, really, to be the descendants of such people or more better, to have in our heart and mind all these theories, knowledge and inventions which so easily had been inherited to us and to the whole world.

- That's the reason I dream... I dream for an ancient Greek mathematical museum like yours. A place to be visited and admired by everybody; a place where might all the great mathematicians be, gathered all together in a huge building, giving an example of wisdom and offer to the humanity and civilization.
- You have Art-museums, archaeological ones, of physics, historical treasures...
- But we haven't a mathematical museum on our planet and I wish it had!

Because every piece of knowledge and every mathematical theorem is like a shining star in the sky at night!

It's like you, my good star that you have brought me here to visit this wonderful state! Thank you!

Eleni, every night, before going to sleep, looks at the stars at night and particularly, she looks at a certain star that lights on and off, shining in the sky!

It's her own star, the ancient Greek star and she always has a dream and makes the same wish! An ancient Greek mathematical museum on Earth! And all this, from that night, the night she had visited the state of the ancient Greek mathematical museum! What do you say?

Would her wish be fulfilled?

16.

Longitude and latitude and International time
A birthday present

Little Lily got up to her room, passing two steps at a time; she ran to sleep. She was tired and happy after today's party for her birthday. All her friends had come to celebrate with her the day of being twelve years old. What a dance, songs and shouts! What sweet presents!

However, now this nice festivity was over, she wanted to sleep but she couldn't because she was recalling her father's words: "Our present, sweet Lily, will be a trip to Great Britain, that is, England! Your mother and I, know how much you want to meet your friend Sally who lives in London. So, we have arranged to travel to London the day after tomorrow, by aeroplane, of course"!!

Lily was very happy for this special present. Her wish was fulfilled. She had wished meeting Sally on her birthday.

She wanted to meet her little friend Sally a long time ago. They were pen friends. They were writing letters regularly to each other describing their countries,, school and school classes. At last, they will meet in a couple of days.

Having in mind all this, she looked at the earth-globe on her desk to find out where was London located. She stood up and spinned it. Then she observed that Earth is divided into horizontal zones . The central line is called equator and the parallel lines are called parallels.

However, it is also divided into vertical lines and the most significant one passes through the observatory of Greenwich and it is called prime-meridian. All the rest lines are called meridians. It's very interesting to go there where the meridian is on 0o.... how happy my little English friend will be my friend with the long fair hair and the blue, clever eyes! At once, she lied on her bed and started dreaming, looking at the earth-globe on her desk.

How will she look like? Like her photo? Will she have freckles on her face? Will she be short-sighted?

I'm from different country. What impression shall I make to her? But is it important?

She is my beloved friend and I want to meet her very much even if we come from different countries. We'll go together to visit several sights of the city, to have fun, sing and as Lily thinks all this... it is dawn at last!! They all get ready, arrive at the airport, check in and get on board it was a four-hour trip, from Cyprus to England and when they arrived they switched to the local time.

Then Lily asked her father:

- Dad, why have we put our watches backwards?

- As you have learnt at school the globe is divided into 24 time-zones.

Whenever we move from one zone to another we have to change our watches accordingly because moving eastwards from Greenwich you have to add hours whereas moving westwards you subtract hours.

- Here is the huge Heathrow airport, we are landing, Lily shouts out excitedly.

She takes a glimpse from the window and she can see a new world, a new country!

Getting off the plane, they move among the crowd searching for Sally.

Soon, Lily sees Sally in front of her. She looked exactly the same, like the one she had imagined: tall and thin, with long, fair hair in pig-tails tied with two red ribbons. She was wearing a red coat and boots.

Her father and little brother Tom were standing next to her. They saw her and welcome her to once. In a while Mr. Brown, Sally's father, was driving them home by his car.

It was a traditional English house, big and beautiful. Cathrene, Sally's mother was waiting for them at the door to welcome them with affection. It was exactly four o'clock; tea-time with traditional, homemade, English biscuits. The English are famous for their accuracy and Big-Ben! recalled Lilly.

After having a rest, they were talking happily about their impressions from their trip to London, about the houses or English people! Cathrene suggested to visit the zoo, at Retzens park in London, the next day. The two girls were excited about it. However, Mr. Brown told them, they had to go there by bus as the parking was a difficult thing at the centre of London.

Therefore, they took the bus the next day to visit the zoo, It was a red double deck and as it was crossing several streets, they were looking at the buildings, the green places and the trees afar. The sky was cloudy and it was going to rain.

They were passing the famous Trafalgar square and the Oxford street where they decided to get off the double-deck to go shopping! They wanted so many things; clothes, shoes, bags et.c.

Later on, passing the Piccadilly circus by bus, they arrived at the Retzens' Park, at the zoo. After spending some time looking at the animals, they left, to have lunch!

- We'll visit the planetarium, tomorrow, to look at the planets and stars. We'll also visit madam Tusseaud's museum.
- Is it there where the wax figures look like the real human beings? asks Lilly.
- Yes, it is there.
- Fine.

The time was passing by and Mr. Brown said with anxiety:

- The last double deck is leaving in few minutes.
- Well, let's go to catch it, said Lilly's father.
- Look, it's leaving! We didn't catch it!
- How will we go home now?
- We'll use the longitude and latitude, said Lilly!
- We can't use them for small distances. They are usually used for villages or cities because they are pair of number that define a spot on the surface of Earth.
- Yes, father, we have learnt it at school, said Sally.
- The first number shows how far we are from the equator either south or north and the second one how far we are from the prime meridian westwards or eastwards. Isn't so, dad? says Lilly.
- Yes, it is so! He answers.
- Don't be afraid, girls, we'll walk till the end of the park, then we'll take a taxi for the central bus-stop and we'll return home by the double-desk.
- What are we waiting for? Let's go.

They've arrived home after a long time. They were very tired. The night was falling and so they went to bed.

Suddenly the alarm-clock rang. Lilly opened her eyes with surprise.

- I've just gone to sleep.

However, her mother and father entered the room very happy.

- Are you ready for our trip, tomorrow?
- What trip? but where are we? Where is Sally?
- Sally?
- Where am I?
- Sally will be waiting for us, tomorrow, Lilly; said her father.
- Come on, get up! We have to get our suitcases ready, my girl, said her mother.
- Our trip hasn't started yet!
- Really? I thought I was already there!
- It was a dream, said Lilly.
- Yes, it was a dream that is going to come true, said her father and mother laughing and embracing their daughter.

17.

Factorial
The puzzle of knowledge of the green dragon

Travelling to China, far away, to the land of green meadows, thick forests and beautiful gardens, there, where the clear waters of the waterfalls overflow to the rivers that cross the land like calm quiet streams, somewhere there in a small village, Ye lives. He is a little boy, with short black hair and beautiful eyes, like almond! Ye, goes to the rice fields with his parents every weekend and helps to the seed or harvest of rice according to the season.

The farmer's life is difficult, especially of the Chinese farmer! He occupies himself all day in the rice-fields, sink in the mud soil of them, trying for a better cultivation of the rice, especially during the monsoons period.

However, it's also difficult to be a farmer's child like Ye, who works hard to earn his living but he can also study the nature and be trained in Tai-tsi like all Chinese, to be physically and mentally healthy.

Well, today is the day before New Year's Day and Ye has just come from his training lesson.

- Father, mother (he bows in front of them)

Chinese, you know, bow to their parents to show respect.

- My son, take your shoes off and come to have lunch.
- Yes, father in a minute, to wash my hands.

He put on his slippers and kneels on the floor over cushions, with his parents.

- We'll have chicken with rice for lunch, today, said his mother.
- My favourite, added Ye, with satisfaction!

They started eating their food. Chinese eat their food with sticks, which was in small bowls, on a tray made of straw and drank water from small glasses.

Later on ...

- Will you come with us to pick some flowers and finish the preparations for the festivity?
- Of course, I'll, mother, the New Year's Day is so important! It's a festivity of spring, of flowers; we hope to have good luck, health and peace in the New Year.
- And a good rice-crop, added his father. The last few years wasn't so good.
- You are right, father
- Well, let's go.

They walk down the footpath slowly-slowly picking flowers and putting them in baskets.

- While picking some flowers Ye saw something wooden behind a bush. It was a small, wooden object, dirty from soil.

Full of curiosity and without much thinking, he took it in his hands and clean it.

It was a wooden triangle with writing on it. He cleans it better, to read it "Take me at the hollow of the old tree next to the green river".

- It's very strange. What does this mean? I'll ask my father.
- Father, look what I found!
- Where do you find it, my child?
- There, behind the bush with the flowers. What does it mean?
- It seems like a piece of a puzzle. In the old days there were puzzles with directions. It must be an old piece. What is written on it?
- The writing says to take it in the hollow of an old tree next to the green river. What shall I do?
- You have to take it! Besides, it isn't far from here and as it is something old, it may be something important or a message...
- Fine father. I go immediately. However I didn't help you much.
- Never mind, another time. This piece of puzzle may hide something you must see...
- You are right!

So, he walks at once (with a quick step quickly through footpaths and bushes towards the river... the green river...

When he arrives, he finds another six children there with a wooden puzzle in their hands.

- Who are you? What are you doing here? Ye asked with surprise.
- I'm Tson and accidentally I found a square wooden puzzle.
- Really?
- I'm Han and I found a parallelogram wooden puzzle.
- My name is Tan and I also found a big equal triangle.
- And I, Gram, I have another one big equal triangle.
- I'm Kim and I have a small equal triangle.
- I'm Mir and I have also a small equal triangle.

- My puzzle is a middle triangle and it seems to have double surface from the smaller ones said Ye.
- Mysterious things, boys, added Han, holding the plait of his hair, we are all seven...
- And with seven puzzle-pieces, said Mir, It must mean something.
- Yes, what is written on your pieces, asked Kim.
- Mine writes: "Take me to the hollow of the old tree next to the green river".
- The writing is the same, they all shouted at once.
- Well, we must solve the puzzle, said Tan with enthusiasm.
- Come on, let's look inside the hollow of this old tree to see if there is something that can help us to join the puzzle.
- But we can't all get inside, interrupted Gram.
- It's very dark hollow and a bit narrow. Only a little child can go inside.
- You mean, like Ye? He is the smallest of all, said Mir smiling.
- Come on, Ye, go inside and look what is in there.
- Ok, I'll try not to be scared.
- We'll watch you.

Ye goes into the hollow carefully and finds an old wooden box. He takes it and goes out of the hollow.

- Let's clean it and put the seven pieces on it.

When they put them together, a square was formed and behind it, in Chinese the word dragon.

Therefore, they decided to form a dragon with the seven puzzle pieces.

After a while, they had done it! But all of a sudden a strong voice was heard.

- "You will be rewarded for your courage and determination!"
- Who is there? Shouted the children with fear.
- Don't be afraid. I'm the spirit of the green dragon and by the coming of the New Year, I'll give you a present because you have solved the puzzle of knowledge.
- But, we can't see you.

- Never mind. I can see you! I know that the last years, your families didn't have a good crop of rice and I can help you, if you do what I'll tell you to do. If you follow my instructions and come here again holding hands around this old tree and shout all together thrice "green dragon" dancing the dragon's dance, I'll have a surprise for you! Do you agree?
- Listen to me carefully. There are seven boxes here with the best seeds of rice inside. You can take a box each of you, and return home at your place. There you must find friends that all together with yourselves must be one less than the ones you are now.
- That's six!
- Yes, and you must tell them to take the six smaller boxes and return also to their places; to find friends thus all together with their themselves, be one less than they were before. How many will they be?
- I found it! Five!
- Right, with the same way those five will take from one box and they will find another and the other in turn, they will get the smaller boxes of their and they will find others till they will be alone. There, they must open the little bags with the rice-seeds and plant them there, in the rice fields, where there isn't any rice.
- It isn't difficult. We, the seven children, must do groups of six, every group must do groups of five, groups of four, three and two.
- This is a multiplication!
- You are right, children. This is a multiplication of many numbers and if the first number is very big, more numbers will be multiplied.
- That is, if I am right, said Ye, we, the seven, we are number (7) seven and the groups of six children we'll make, they will be number six.
- This is apparently clear! All the bags with the rice-seeds are 7X6X5X4X3X2X1
- Yes and I made the calculations and all the little bags in the boxes are 5040.
- Bravo, children, you are all very clever.
- Go now and when you finish, don't forget to come here, I'll be wait for you.
- We thank you, good dragon!!

They all run back to their families at once, find other children and do exactly what the dragon had said.

Then, they go to their rice-fields to plant the seeds.

After a long time, the children met again.

- Eh, Have you done all that the dragon said, asked Ye,

- Yes, Yes, but we've been very tired, added Kim.
- We are all very tired but we must go because it's getting dark.
- Have you brought the lamps to light our way? Asked Mir.
- Yes, Yes, we've also told our parents not to worry if we are a bit late.
- Fine.
- We set off for the green river and I hope to be on time. It's also the eve before the New Year's Day and we'll go to watch the fireworks and dance the dragon's dance.
- You're talking, right. There will also be colourful, kites many dances and songs.
- We must hurry, said Tan.

They hurried through the footpaths and reached the old tree, next to the green river.

- Come on, my friends. Put the lamps around the tree to see better and hold hand by hand.
- OK, Ye! We are ready and shout all together while dancing: "Green dragon"
- "Green dragon!"
- "Green dragon!"

At once the tree started to move and a warm breeze started blowing Colourful rays were coming out of the hollow of the tree, forming a green bright dragon with the wings of a bat and a head of a lion.

It stood in front of the children on his four, short legs decorated with Chinese objects its was bright!

- Ah, Ah, the dragon is alive, shouted the children.
- Don't be afraid. This is the surprise I've promised and now the bigger one of your life! Get on my back all of you and hold tightly.

With a strong movement, it flew in the air by the clouds with the children on its back.

- How nice! It's fantastic, it's marvellous; the children were shouting with excitement.
- We'll fly down, to see the whole china that celebrates for the New Year's Day. The clock at Tien–an-men – stroke twelve.
- It's the year of the dragon, my year. You'll have a rich rice-crop from today. The silk from cloth will be better and softer. The Chinese vases Mink will be nicer. The museum of the forbidden city will be prettier like all the other monuments of China.
- Great!
- And more important, you'll have peace, love and prosperity, because the year of dragon will be the prettiest and the luckiest.

- Look, children, says Ye we've dressed in red with green ribbons, in a magical way.
- Yes, children. The red is the colour of Luck and the green because you're on my back and I'm green!!

Look down, the fireworks have lightened and the festivities for the New Year's Day and Spring have started too. Let's go and see all the decorated cities and later on I'll take you home to continue the celebrations, said the dragon.

And so it happened! They visited all the cities with dragon's company and they had a lot of fun!

Before taking them home, the dragon had given them a piece of puzzle for rememberance and asked them to call him every New Year's Day to bring them good luck!!

In China, the dragon symbolizes Luck and prosperity, you know!!

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